REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
February 1960

2. REPORT TYPE

3. DATES COVERED (From - To)
February 1960

4. TITLE AND SUBTITLE
The Electrostatic Potential due to a Point Charge in the Presence of a Conducting Sphere and an Infinite Ground Plane

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

5d. PROJECT NUMBER
DA 5N06-01-010

5e. TASK NUMBER

5f. WORK UNIT NUMBER

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Diamond Ordnance Fuze Laboratories
Washington 25, DC

8. PERFORMING ORGANIZATION REPORT NUMBER
TR-804

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
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15. SUBJECT TERMS
electrostatic potential, ground plane, Lambert series

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
Unclassified	Unclassified	Unclassified	UU	43	Larry Merkle

19b. TELEPHONE NUMBER (*Include area code*)
301-394-0941

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18

TR-804

TR-804

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Clyde Morrison

Nick Karayianis

1 February 1960

DIAMOND ORDNANCE FUZE LABORATORIES
ORDNANCE CORPS ● DEPARTMENT OF THE ARMY

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DA5N06-01-010
ORD Proj. TN3-9109
DOFL Proj. 10210

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CONTENTS

	Page
ABSTRACT	5
1. INTRODUCTION	5
2. THEORY	5
3. RESULTS AND DISCUSSION	19
REFERENCES AND BIBLIOGRAPHY	20
APPENDIX A. Tabulated Values of F(1, x)	21
APPENDIX B. Inversion Formulas for Series Expansion	22

ABSTRACT

The solution of a particular electrostatic potential problem involving a conducting sphere and infinite ground plane in the presence of a point charge is discussed. Use is made of the conventional imaging method to obtain the solution in terms of Lambert series. It is shown how some of the resultant series may be combined and expressed as contour integrals in the complex plane, and how, by a suitable deformation of the contour, the series may be transformed into an extremely rapidly convergent series. The contour method is used to obtain very simply an asymptotic expansion of part of the solution for a limiting case of the problem which was obtained very laboriously by Schlömilch.

1. INTRODUCTION

The problem in electrostatics of determining charges and potentials on conducting spheres and ground planes in the presence of each other and in various combinations has been of interest to physicists for many years. The problems, in general, are most easily solved by the method of images, to which there are many standard references (ref 1).* The solutions for the charges and potentials on the spheres are in the form of sums of Lambert series (ref 2) with coefficients a_n of the form $\lambda^{n/2}$, where λ may be positive or negative.

The Lambert series had been expressed in several forms; however, none were good for computing results when two of the conducting bodies are very close. In 1860, Schlömilch (ref 3) devised a method which was later extended by Russell (ref 4) by which the very close case can be expressed in an asymptotic series which is more accurate as the bodies move closer together.

Even with the contributions of Schlömilch and Russell, there was still a need for more improvement in the computational methods. The rapidly convergent forms for the Lambert series have individual terms which require laborious computation in themselves and for this reason are no more desirable for computation than a more slowly convergent Lambert series which has relatively simple individual terms.

In many problems, two of the Lambert series in the solution combine to form a single series which is summed from $-\infty$ to $+\infty$. This series may be expressed as a contour integral in the complex plane (ref 5). A proper adjustment of the path of integration yields a single series which is so rapidly convergent that any more than the first two terms are rarely required.

Below, a specific potential problem involving a conducting sphere and plane, and a point charge is handled in some detail. The standard

* References are listed on page 20.

imaging methods are used to obtain the solution, and the more powerful methods described above are applied to reduce the solution to a more usable form.

2. THEORY

In the theory of electrical images it is shown that a conducting sphere of radius a, (figure 1) in the presence of a positive point charge q located at distance d from its center will be an equipotential surface if a negative image charge of magnitude $\frac{a}{d}q$ is placed on the axis joining the charge and sphere at a distance $\frac{a^2}{d}$ from the center of the sphere (ref 1). This combination causes the sphere's surface to be at zero potential. If we wish the sphere to be at any other potential, we may place an additional charge at the center so that the potential of the sphere is this charge divided by the radius. Similarly, the image which keeps an infinite plane an equipotential in the presence of a point charge is a charge of equal magnitude and opposite sign placed an equal distance below the plane.

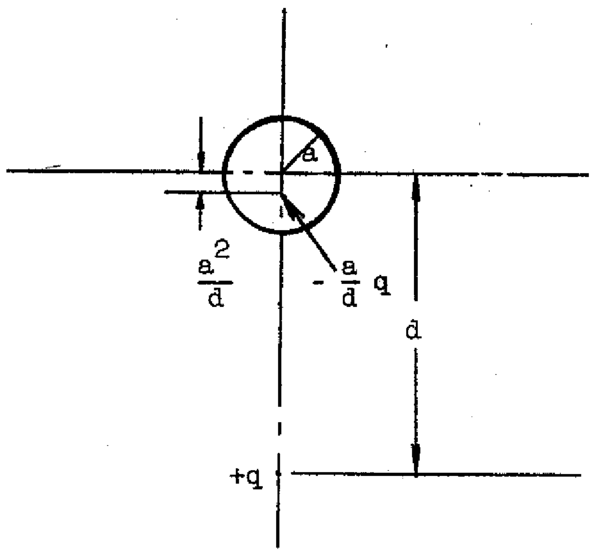

Figure 1

When we consider the problem of a charged sphere and a plane as shown in figure 2, we find that an infinite number of images are necessary to meet the conditions that the plane and sphere are equipotential surfaces. The charge Q at the center of the sphere images in the plane as -Q to keep the plane at equipotential. Now, however, the sphere in the presence of -Q is not at equipotential until -Q is imaged back in the sphere. This image charge makes the plane nonequipotential until it images back in the plane, and so the process continues. The result is an infinite series of charge images. The potential of the sphere is determined by the charge at its center, i.e., $V_s = \frac{Q}{a}$, where a is the sphere

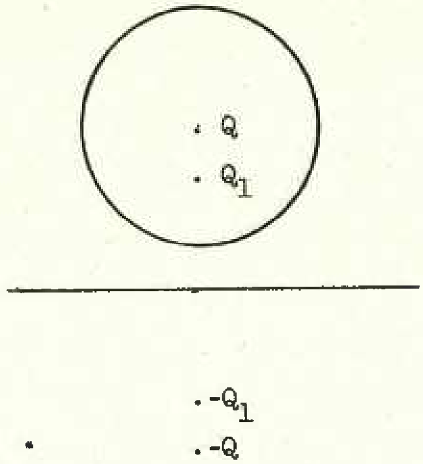

Figure 2

radius, and the total charge on the sphere is the sum of the center charge and all the image charges in the sphere. The plane, of course, stays at zero potential. Similarly, the problem of a point charge located between the sphere and plane, as shown in figure 3, will give rise to an infinite number of images. For simplicity, the series of images is broken into two parts as shown in figure 4a and figure 4b. The problem that is treated here is a charged conducting sphere in the presence of an external point charge and an infinite ground plane.

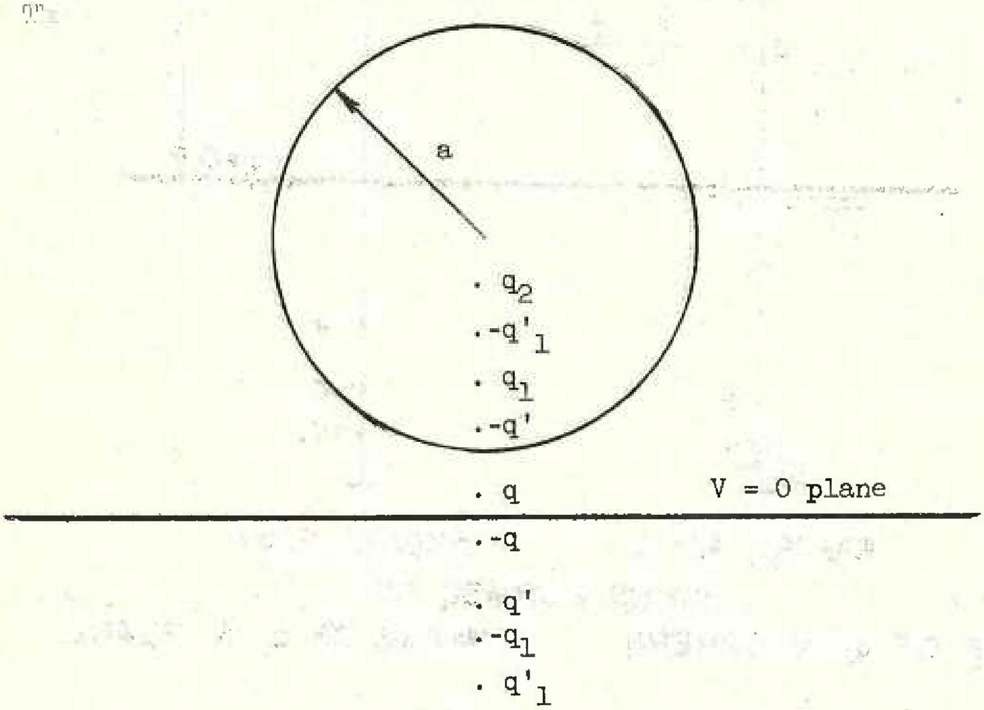

Figure 3

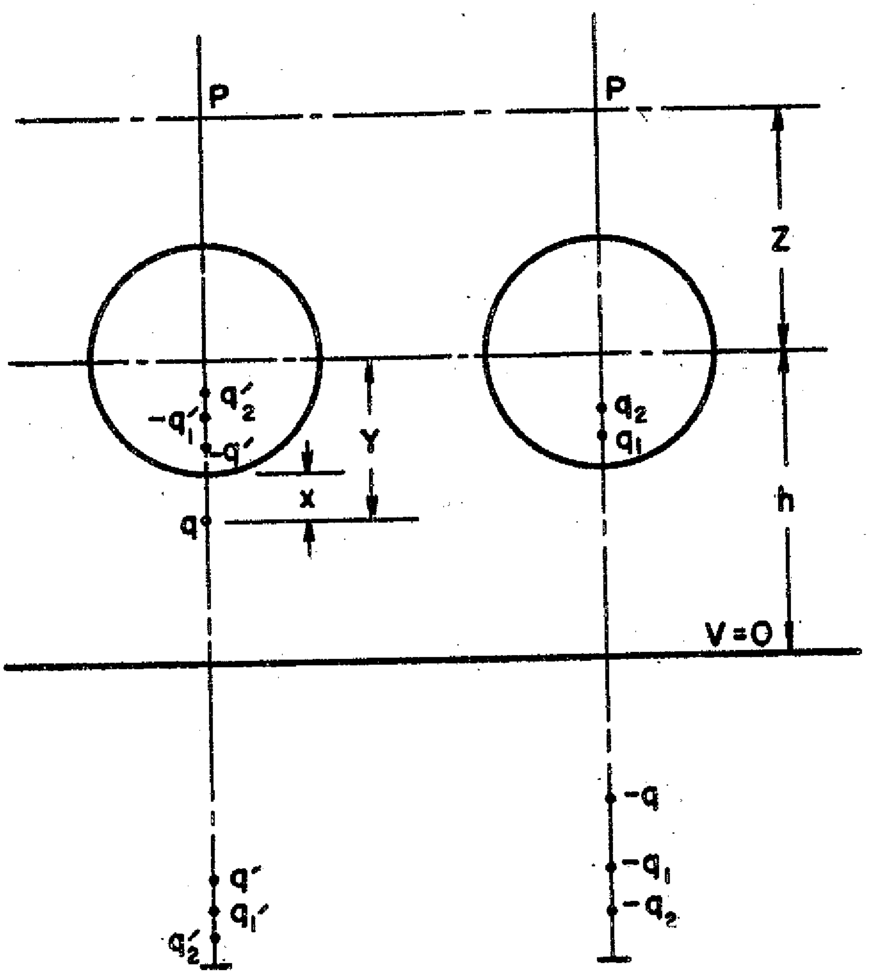

FIGURE 4(a) FIGURE 4(b)

SERIES FORMED BY

IMAGE OF q IN SPHERE IMAGE OF q IN PLANE

Figure 4

The first series to be treated is the one resulting from the center charge of the sphere as shown in figure 2. The distance from sphere center and magnitude of the various charges can be obtained from the simple case considered in figure 1. They are given in the following table together with the potential due to each charge at the point P located at distance Z from the sphere center as shown in figure 4.

TABLE I

Magnitude of charge	Distance from center of sphere	Potential at P
Q $Q_1 = \frac{a}{2h} Q$	0 $\frac{a^2}{2h}$	$\frac{Q}{Z}$ $\frac{Q_1}{Z + \frac{a^2}{2h}}$
$Q_2 = \frac{a}{2h - \frac{a^2}{2h}} Q_1$	$\frac{a^2}{2h - \frac{a^2}{2h}}$	$\frac{Q_2}{Z + \frac{a^2}{2h - \frac{a^2}{2h}}}$
$Q_n = \frac{a}{D_n} Q_{n-1}$ $D_n = 2h - \frac{a^2}{D_{n-1}}$	$\frac{a^2}{D_n}$	$\frac{Q_n}{Z + \frac{a^2}{D_n}}$

The last relation for Q_n can be used to express any change in terms of Q or

$$Q_n = \frac{a}{D_n} \frac{a}{D_{n-1}} \cdot \cdot \cdot \frac{a}{D_1} Q.$$

If we let $\frac{D_n}{a} = R_n$ and $\frac{h}{a} = \mu$ then we have $R_n = 2\mu - \frac{1}{R_{n-1}}$ with $R_1 = 2\mu$.

This equation becomes simplified if we let

$$R_n = \frac{U_{n+1}}{U_n} \text{ or } \frac{U_{n+1}}{U_n} = 2\mu - \frac{U_{n-1}}{U_n}$$

then

$$U_{n+1} = 2\mu U_n - U_{n-1} \tag{1}$$

This equation is a standard second order difference equation. The solution of this equation can be found by assuming $U \propto \beta^n$ where β is to be determined. If this is substituted into equation 1, we have

$$\beta^{n+1} - 2\mu\beta^n + \beta^{n-1} = 0$$

which has the nontrivial solutions: $\beta = \mu \pm \sqrt{\mu^2 - 1}$. (2)

We then chose

$$U_n = A_1\beta_1{}^n + A_2\beta_2{}^n$$

where $\beta_1 = \mu + \sqrt{\mu^2 - 1}$

$\beta_2 = \mu - \sqrt{\mu^2 - 1}$, $\beta_1\beta_2 = 1$,

and the constants are determined from the condition $R_1 = \dfrac{U_2}{U_1} = 2\mu$.

Since just the ratio $\dfrac{U_{n+1}}{U_n}$ is required, this one boundary condition is sufficient. The resulting solution is:

$$R_n = \frac{\beta_1{}^{n+1} - \beta_2{}^{n+1}}{\beta_1{}^n - \beta_2{}^n} . \tag{3}$$

Now since $Q_n = \dfrac{a}{D_n}\ \dfrac{a}{D_{n-1}} \cdots \dfrac{a}{D_1}\ Q = \dfrac{1}{R_n}\ \dfrac{1}{R_{n-1}} \cdots \dfrac{1}{R_1}\ Q$,

then

$$Q_n = \frac{\beta_1 - \beta_2}{\beta_1{}^{n+1} - \beta_2{}^{n+1}} Q. \tag{4}$$

The total charge on the sphere due to this series of images is

$$Q_Q = Q + Q_1 + Q_2 \cdots\cdot$$

$$= Q + Q \sum_1^{\infty} \frac{\beta_1 - \beta_2}{\beta_1{}^{n+1} - \beta_2{}^{n+1}}$$

$$Q_Q = Q(\beta_1 - \beta_2) \sum_0^{\infty} \frac{1}{\beta_1^{n+1} - \beta_2^{n+1}} ,$$

and since $\beta_1 = \frac{1}{\beta_2}$

$$Q_Q = Q(\beta_1 - \beta_2) \sum_1^{\infty} \frac{\beta_2^{n}}{1 - \beta_2^{2n}} . \qquad (5)$$

The potential at P due to the series of images inside the sphere is

$$V_{Q_s} = \frac{Q}{Z} + \sum_1^{\infty} \frac{Q_n}{Z + \frac{a^2}{D_n}} .$$

Now

$$\frac{Q_n}{Z + \frac{a^2}{D_n}} = \frac{Q}{a} \frac{\beta_1 - \beta_2}{z + \beta_2} \left[\frac{1}{\beta_1^{n+1} - \frac{z+\beta_1}{z+\beta_2} \beta_2^{n+1}} \right]$$

and

$$V_{Q_s} = \frac{Q}{a} \frac{\beta_1 - \beta_2}{z + \beta_2} \sum_1^{\infty} \frac{\beta_2^{n}}{1 - \frac{z+\beta_1}{z+\beta_2} \beta_2^{2n}} \qquad (6)$$

where $z = \frac{Z}{a}$.

If we set

$$F(\lambda\ \beta_2) = \sum_1^{\infty} \frac{\beta_2^{n}}{1-\lambda\beta_2^{2n}} ,$$

then

$$Q_Q = Q(\beta_1 - \beta_2)\ F(1,\beta_2)^{*} \qquad (7)$$

and

$$V_{Q_s} = \frac{Q}{a} \frac{\beta_1 - \beta_2}{z + \beta_2} F \left(\frac{z + \beta_1}{z + \beta_2} , \ \beta_2 \right) . \qquad (8)$$

* A Table of $F(1,\beta_2)$ with the first three differences has been included in Appendix A.

The potential at P due to the images in the plane is

$$V_{Q_p} = - \frac{Q}{Z + 2h} - \sum_{1}^{\infty} \frac{Q_n}{Z + 2h - \frac{a^2}{D_n}}$$

and

$$\frac{Q_n}{Z + 2h - \frac{a^2}{D_n}} = \frac{Q}{a} \frac{(\beta_1 - \beta_2)}{z + \beta_1} \left[\frac{1}{\beta_1^{n+1} - \frac{z + \beta_2}{z + \beta_1} \beta_2^{n+1}} \right]$$

then

$$V_{Q_p} = - \frac{Q}{a} \frac{\beta_1 - \beta_2}{z + \beta_1} F \left(\frac{z + \beta_2}{z + \beta_1}, \beta_2 \right) \tag{9}$$

The potential at P due to all the images is then

$$V_Q = \frac{Q}{a} \frac{(\beta_1 - \beta_2)}{z + \beta_2} \left[F \left(\frac{z + \beta_1}{z + \beta_2}, \beta_2 \right) - \frac{z + \beta_2}{z + \beta_1} F \left(\frac{z + \beta_2}{z + \beta_1}, \beta_2 \right) \right] \tag{10}$$

A table similar to Table I can be made for the series shown in figure 4a. See Table II on following page.

The same difference equation is obtained by the substitution previously used, the only difference being the constants in the solution. If the condition

$R_o = y$, $(y = \frac{Y}{a})$, is used then

$U_n = \beta_1^n - \frac{y-\beta_1}{y-\beta_2} \beta_2^n$ or letting $\gamma' = \frac{y-\beta_1}{y-\beta_2}$

$U_n = \beta_1^n - \gamma' \beta_2^n$ where again $R_n = \frac{D_n}{a} = \frac{U_{n+1}}{U_n}$.

TABLE II

Magnitude of charge	Distance from center of sphere	Potential at P
$q'_o = \frac{a}{Y} q$	$\frac{a^2}{Y}$	$\frac{q'_o}{Z + \frac{a^2}{Y}}$
$q'_1 = \frac{a}{2h - \frac{a^2}{Y}} q'_o$	$\frac{a^2}{2h - \frac{a^2}{Y}}$	$\frac{q'_1}{Z + \frac{a^2}{2h - \frac{a^2}{Y}}}$
$q'_2 = \frac{a}{2h - \frac{a^2}{2h - \frac{a^2}{Y}}} q'_1$	$\frac{a^2}{2h - \frac{a^2}{2h - \frac{a^2}{Y}}}$	$\frac{q'_2}{Z + \frac{a^2}{2h - \frac{a^2}{2h - \frac{a^2}{Y}}}}$
$q'_n = \frac{a}{D_n} q'_{n-1}$	$\frac{a^2}{D_n}$	$\frac{q'_n}{Z + \frac{a^2}{D_n}}$

$$\text{where} \quad D_n = 2h - \frac{a^2}{D_{n-1}}$$

The total charge inside the sphere due to these images is

$$Q_{q's} = -q'_o - q'_1 - q'_2 - \ldots .$$

and as before

$$Q_{q's} = -q(1-\gamma') \sum_{1}^{\infty} \frac{\beta_2^{\,n}}{1-\gamma'\beta_2^{\,2n}} = -q(1-\gamma')\, F(\gamma',\beta_2). \qquad (11)$$

The potential at P due to this series of charges can be calculated by the same method used in deriving equation 5 and is

$$V_{q'_s} = - \frac{q}{a} \frac{1-\gamma'}{z+\beta_2} F(A,\beta_2), \text{ where } A \equiv \gamma' \frac{z+\beta_1}{z+\beta_2} . \tag{12}$$

The potential at P due to the image charges in the plane is

$$V_{q'_p} = \frac{q}{a} \frac{1-\gamma'}{z+\beta_1} F(B,\beta_2) \text{ where } B = \gamma' \frac{z+\beta_2}{z+\beta_1} \tag{13}$$

The total potential at P due to both series of images is

$$V_{q'} = V_{q'_p} + V_{q'_s} = \frac{q}{a} \frac{(1-\gamma')}{z+\beta_1} \left[F(B,\beta_2) - \frac{z+\beta_1}{z+\beta_2} F(A,\beta_2)\right] \tag{14}$$

The charge and potential due to the two series of images shown in figure 4b can be obtained by the same method used in the two cases given above. The total charge inclosed in the sphere is:

$$Q_{q_s} = q(1-\gamma)\ F(\gamma,\beta_2) \tag{15}$$

where $\gamma = \frac{1}{\gamma'} = \frac{y - \beta_2}{y - \beta_1}$.

The potential due to the image charges in the sphere and the plane is

$$V_q = \frac{q}{a} \frac{(1-\gamma)}{z+\beta_2} \left[F(C,\beta_2) - \frac{z+\beta_2}{z+\beta_1} F(D,\beta_2)\right] - \frac{q}{a} \frac{1}{z+2\mu-y} \tag{16}$$

where $C = \frac{1}{\gamma'} \frac{z+\beta_1}{z+\beta_2}$, $D = \frac{1}{\gamma'} \frac{z+\beta_2}{z+\beta_1}$, and the last term is the potential due to the image of the charge, q, in the plane. The total potential at P due to all the charges and their images is now just the sum of the potentials given by equations 9, 14, 16, and the potential due to the charge itself, $\frac{q}{a} \frac{1}{z+y}$.

The above equations give the potential at the point P if some condition is given to determine Q the charge at the center of the sphere. The two cases we will consider are (I) when Q = 0, i.e., when the sphere is grounded, and (II) when the entire system is neutral, i.e.,

$$Q_Q + Q_{q'} + Q_q = -q.$$

Case I. Grounded Sphere

The condition Q = 0 removes equation 9 from the expression for the potential at P. This potential becomes

$$V' = \frac{q}{a}\left\{\frac{1-\gamma'}{z+\beta_1} F(B,\beta_2) - \frac{1-\gamma'}{z+\beta_2} F(A,\beta_2) + \frac{1-\gamma}{z+\beta_2} F(C,\beta_2) - \frac{1-\gamma}{z+\beta_1} F(D,\beta_2)\right\} + \frac{q}{a}\,\frac{1}{z+y} - \frac{q}{a}\,\frac{1}{z+2\mu-y}, \tag{17}$$

where $D = \frac{1}{A}$ and $C = \frac{1}{B}$ from equation (11), (12), and (16). Now

$$\sum_{-\infty}^{\infty} \frac{\beta_2^{\,n}}{1-A\beta_2^{\,2n}} = F(A,\beta_2) - \frac{1}{A} F\left(\frac{1}{A},\beta_2\right) + \frac{1}{1-A}, \tag{18}$$

then

$$V' = \frac{q}{a}\,\frac{1-\gamma'}{z+\beta_1}\left\{\sum_{-\infty}^{\infty} \frac{\beta_2^{\,n}}{1-B\beta_2^{\,2n}} - \frac{z+\beta_1}{z+\beta_2} \sum_{-\infty}^{\infty} \frac{\beta_2^{\,n}}{1-A\beta_2^{\,2n}}\right\}. \tag{19}$$

Now when the sphere is at a large distance from the plane, $\beta_2 \to 0$ so that the potential is given by $V'_{h\to\infty} = \frac{q}{a}\left[\frac{1}{y+z} - \frac{1}{yz+1}\right]$ which is the potential at P when there is no plane present.

Case II. Neutral System

The condition that the entire system be neutral requires that

$$-q = q\,(1-\gamma)\,F(\gamma,\beta_2) + Q(\beta_1-\beta_2)\,F(1,\beta_2) - q(1-\gamma')\,F(\gamma',\beta_2).$$

This equation then determines the charge at the center of the sphere

$$Q = q\,\frac{(1-\gamma')\,F(\gamma'\beta_2) - 1 - (1-\gamma)\,F(\gamma,\beta_2)}{(\beta_1-\beta_2)\,F(1,\beta_2)}. \tag{20}$$

The potential of the sphere is $\frac{Q}{a}$. Equation (18 can be used to write equation (10) in a simpler form, hence

$$V_Q = \frac{Q}{a} + \frac{Q(\beta_1 - \beta_2)}{a\,(z + \beta_2)} \sum_{-\infty}^{\infty} \frac{\beta_2^{\,n}}{1 - \frac{z+\beta_1}{z+\beta_2}\,\beta_2^{\,2n}} \tag{21}$$

The potential at P becomes

$$V = V_s + \frac{Q}{a}\,\frac{(\beta_1 - \beta_2)}{(z + \beta_2)} \sum_{-\infty}^{\infty} \frac{\beta_2^{\,n}}{1 - \frac{z+\beta_1}{z+\beta_2}\,\beta_2^{\,2n}}$$

$$+ \frac{q}{a}\,\frac{(1-\gamma')}{(z+\beta_1)} \left[\sum_{-\infty}^{\infty} \frac{\beta_2^{\,n}}{1-B\beta_2^{\,2n}} - \sum_{-\infty}^{\infty} \frac{\beta_2^{\,n}}{1-A\beta_2^{\,2n}} \right] \tag{22}$$

where $V_s = \frac{Q}{a}$ and Q is given by equation (20).

The two equations given (22, 19) are good for larger distances from the plane. If the sphere comes close to the plane, it is necessary to invert the series as shown in appendix B to obtain series which converge rapidly enough for practical computation. The results are relatively simple for the case of the grounded sphere or

$$V' = \frac{q}{a}\,\frac{1-\gamma'}{z+\beta_1}\,\frac{\pi}{2\alpha\sqrt{-B}} - \frac{z+\beta_1}{z+\beta_2}\,\frac{\pi}{2\alpha\sqrt{-A}} + \frac{\pi}{a} \left[\sum_{0}^{\infty} \frac{1}{\sqrt{-B}}\,\frac{e^{-(2n+1)\pi^2/\alpha} - \cos\theta_1}{\cos\theta_1 - \cosh\frac{\pi^2}{\alpha}(2n+1)} \right.$$

$$\left. - \frac{z+\beta_1}{z+\beta_2}\,\frac{1}{\sqrt{-A}}\,\frac{e^{-(2n+1)\pi^2/\alpha} - \cos\theta_2}{\cos\theta_2 - \cosh\frac{\pi^2}{\alpha}(2n+1)} \right] \tag{19a}$$

where

$$\theta_1 = \frac{\pi \ln(-B)}{\alpha}, \quad \theta_2 = \frac{\pi \ln(-A)}{\alpha} \quad \text{and } \alpha = \ln \beta_1 .$$

3. RESULTS AND DISCUSSION

In all the results the potential was calculated at the point on the opposite side of the sphere at the same distance from the sphere as the charge i.e., $z = y$. The potential difference between the sphere and the point was computed i.e., $V - V_s$, and this was normalized to the value of $V - V_s$ at infinite distances from the plane. In figure 5 the normalized potential

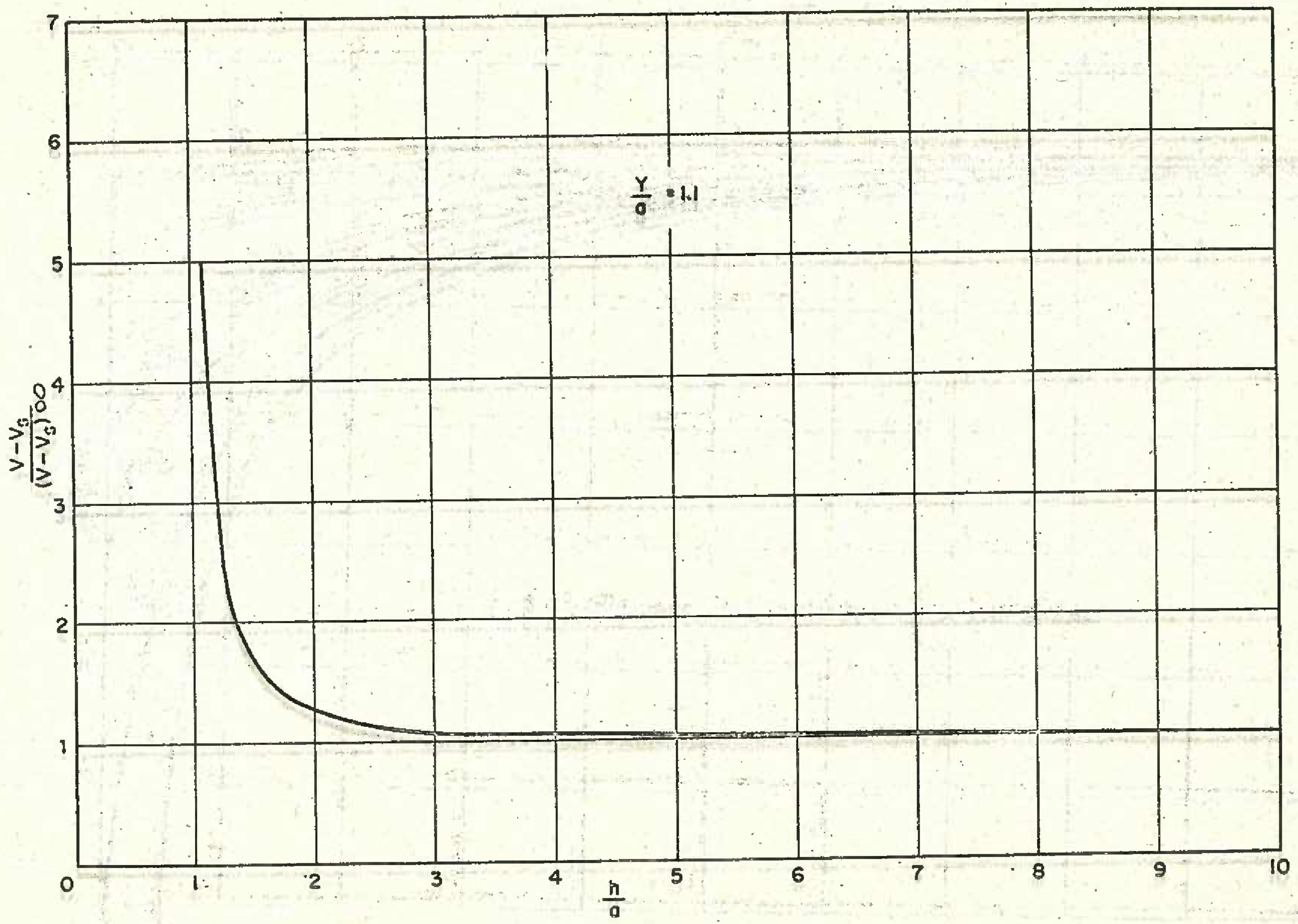

Figure 5. Normalized potential for the case $\frac{Y}{a} = 1.1$

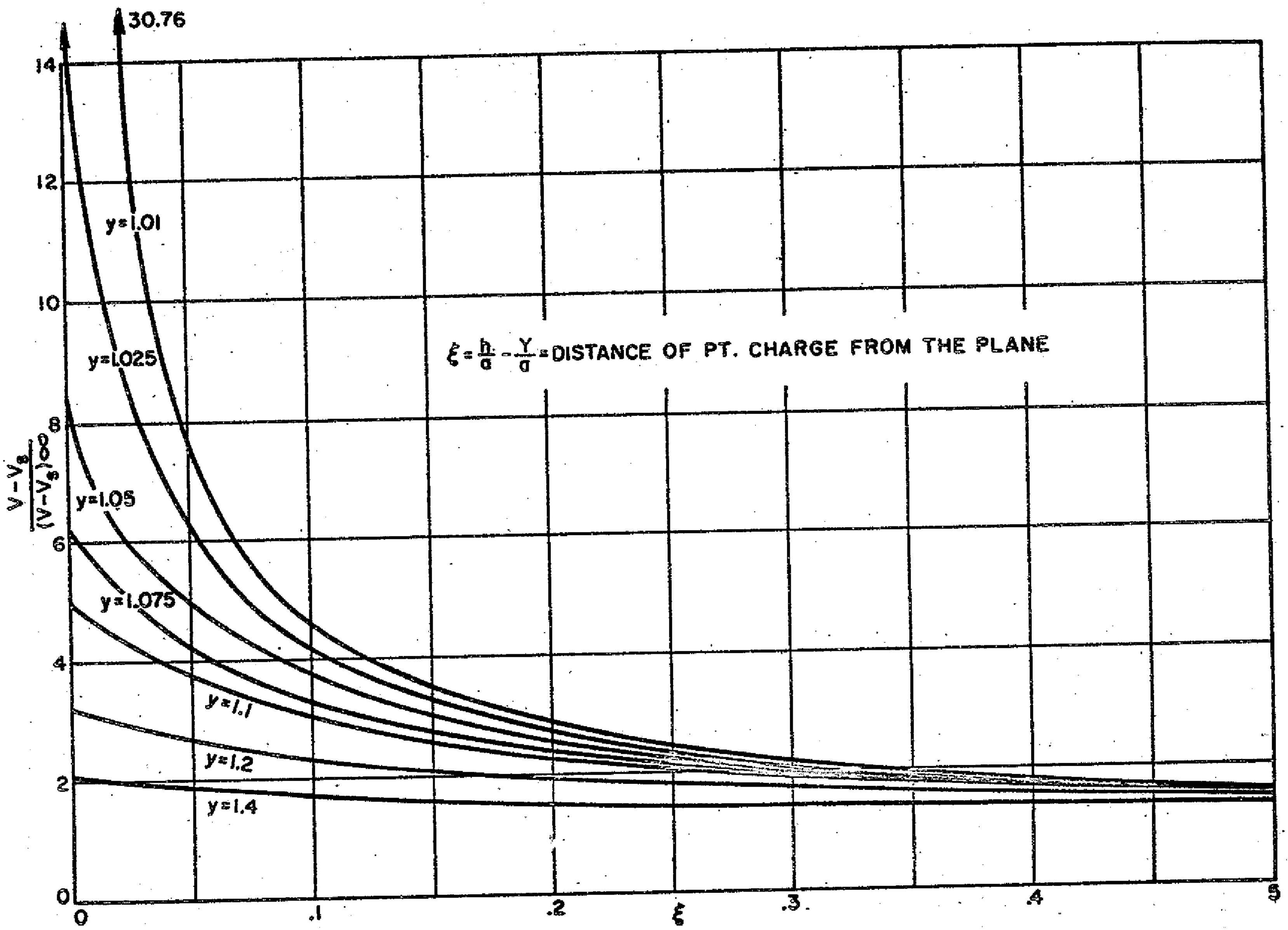

Figure 6. Normalized potentials for several values of $y = \frac{Y}{a}$

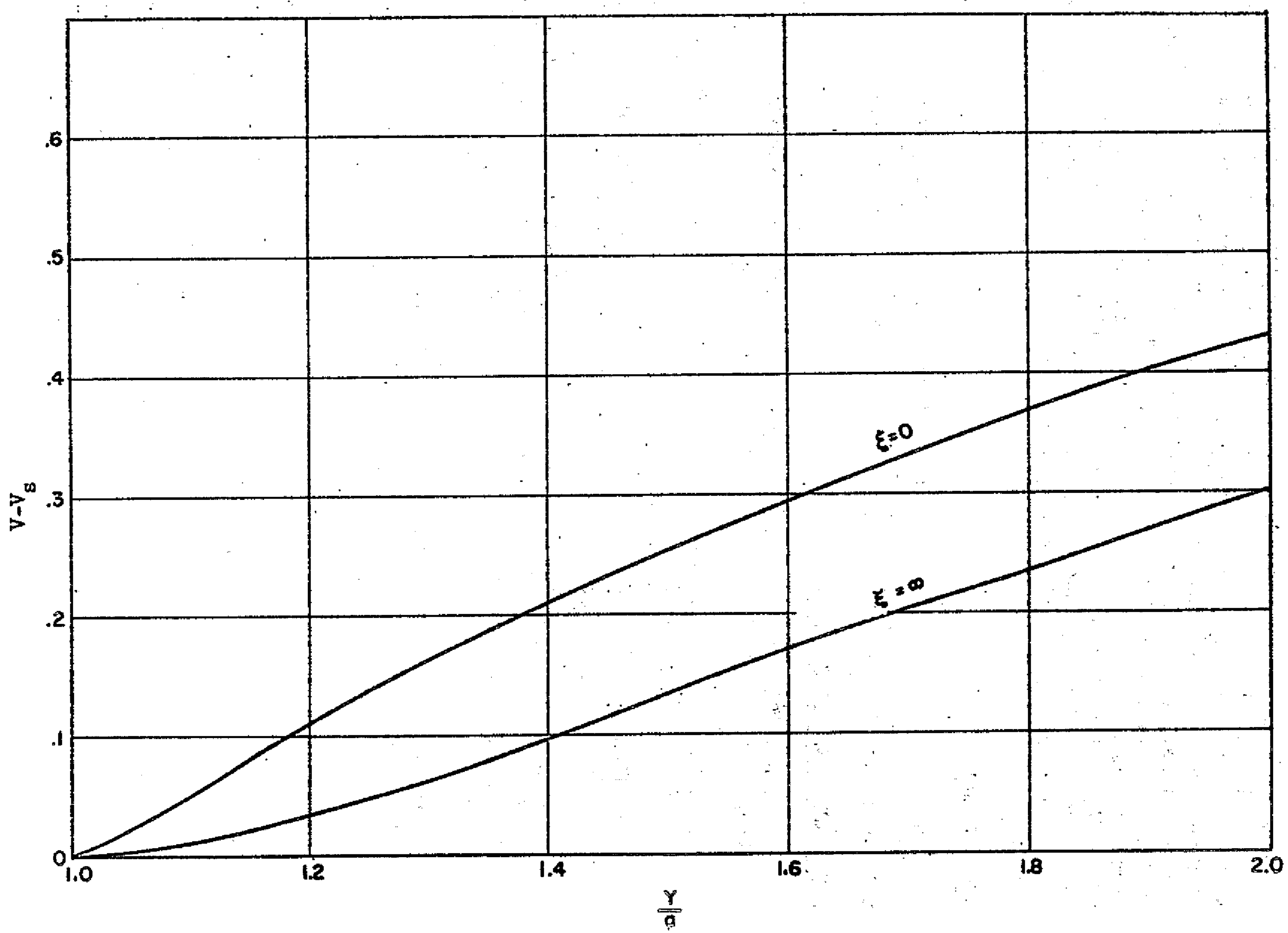

Figure 7. V-V_s at infinity and at contact for q/a unity as a function of $\frac{Y}{a}$

for the case Y/a = 1.1 is plotted versus the distance from the plane to the sphere's center, h/a. At great distances the curve follows an inverse cubic but as the sphere approaches the plane the potential rises much faster. In all the cases computed the results are essentially the same at large distances. In figure 6 the normalized potential for a number of different values of Y/a are plotted versus distance between the charge and the plane. The normalized potential increases rapidly as Y/a becomes small. This is caused by the small value of $V - V_s$ at infinite distances from the plane not by the increase in $V - V_s$ close to the plane. A plot of $V - V_s$ at infinity and at contact for q/a unity is shown in figure 7 as a function of Y/a and shows that the value of $V - V_s$ at infinity decreases more rapidly than when the charge touches the plane for small values of Y/a.

ACKNOWLEDGMENT

The authors wish to thank Henry P. Kalmus for suggesting the problem; Isabelle Arsham, without whose incalculable patience at the computer this paper would have been graphless, and Fan King for the figures.

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APPENDIX A

TABULATED VALUES OF F(1, x)

x	F(1, x)	Δ_1	Δ_2	Δ_3
0.00	0.00000 00000	0.05275 69079	0.00660 84075	0.00199 87624
0.05	0.05275 69079	0.05936 53154	0.00844 71699	0.00220 35883
0.10	0.11212 22233	0.06781 24853	0.01065 07582	0.00273 70204
0.15	0.17993 47086	0.07846 32435	0.01338 77786	0.00350 91800
0.20	0.25839 79521	0.09185 10221	0.01689 69586	0.00462 88066
0.25	0.35024 89742	0.10874 79807	0.02152 57652	0.00627 17010
0.30	0.45899 69549	0.13027 37459	0.02779 74662	0.00873 23824
0.35	0.58927 07008	0.15807 12121	0.03652 98486	0.01252 35835
0.40	0.74734 19129	0.19460 10607	0.04905 34321	0.01857 84416
0.45	0.94194 29736	0.24365 44928	0.06763 18737	0.02868 85544
0.50	1.18559 74664	0.31128 63665	0.09632 04281	0.04652 73275
0.55	1.49688 38329	0.40760 67946	0.14284 77556	0.08027 03311
0.60	1.90449 06275	0.55045 45502	0.22311 80867	0.15012 96943
0.65	2.45494 51777	0.77357 26369	0.37324 77810	0.31358 95199
0.70	3.22851 78146	1.14682 04179	0.68683 73009	0.76998 88800
0.75	4.37533 82325	1.83365 77188	1.45682 61809	2.46195 69250
0.80	6.20899 59513	3.29048 38997	3.91878 31059	13.49993 80485
0.85	9.49947 98510	7.20926 70056	17.41872 11544	
0.90	16.70874 68566	24.62798 81600		
0.95	41.33673 50166	13.39993 70411	10.11793 29922	16.11990 36082
0.96	54.73667 20577	23.51787 00333	26.23783 66004	88.05181 03388
0.97	78.25454 20910	49.75570 66337	114.28964 69392	
0.98	128.01024 87247	164.04535 35729		
0.99	292.05560 22976			
0.9995	8868.79733 5			

APPENDIX B

INVERSION FORMULAS FOR SERIES EXPANSION

The series used in computation of the potential can be converted using complex variables. The method is to multiply the series by a function having only simple poles at the integers involved in the sum and with a residue of $1/2\pi i$ at these poles.

Consider

$$\sum_{-\infty}^{\infty} \frac{x^n}{1-\lambda x^{2n}} \text{ with } x < 1,\ \lambda x^2 < 1, \text{ and } e^{-\alpha} = x.$$

If we multiply the quantity $\dfrac{e^{-\alpha z}}{1-\lambda e^{-2\alpha z}}$ by $\dfrac{1}{e^{2\pi i z}-1}$ and integrate the resulting expression around a contour inclosing only the poles of $\dfrac{1}{e^{2\pi i z}-1}$ along the real axis, we obtain the given sum

$$\sum_{-\infty}^{\infty} \frac{e^{-\alpha n}}{1-\lambda e^{-2\alpha n}} = \int_c \frac{e^{-\alpha z}}{1-\lambda e^{-2\alpha z}} \frac{dz}{e^{2\pi i z}-1}$$

However, the contour we shall choose to convert the sum is shown in figure B1.

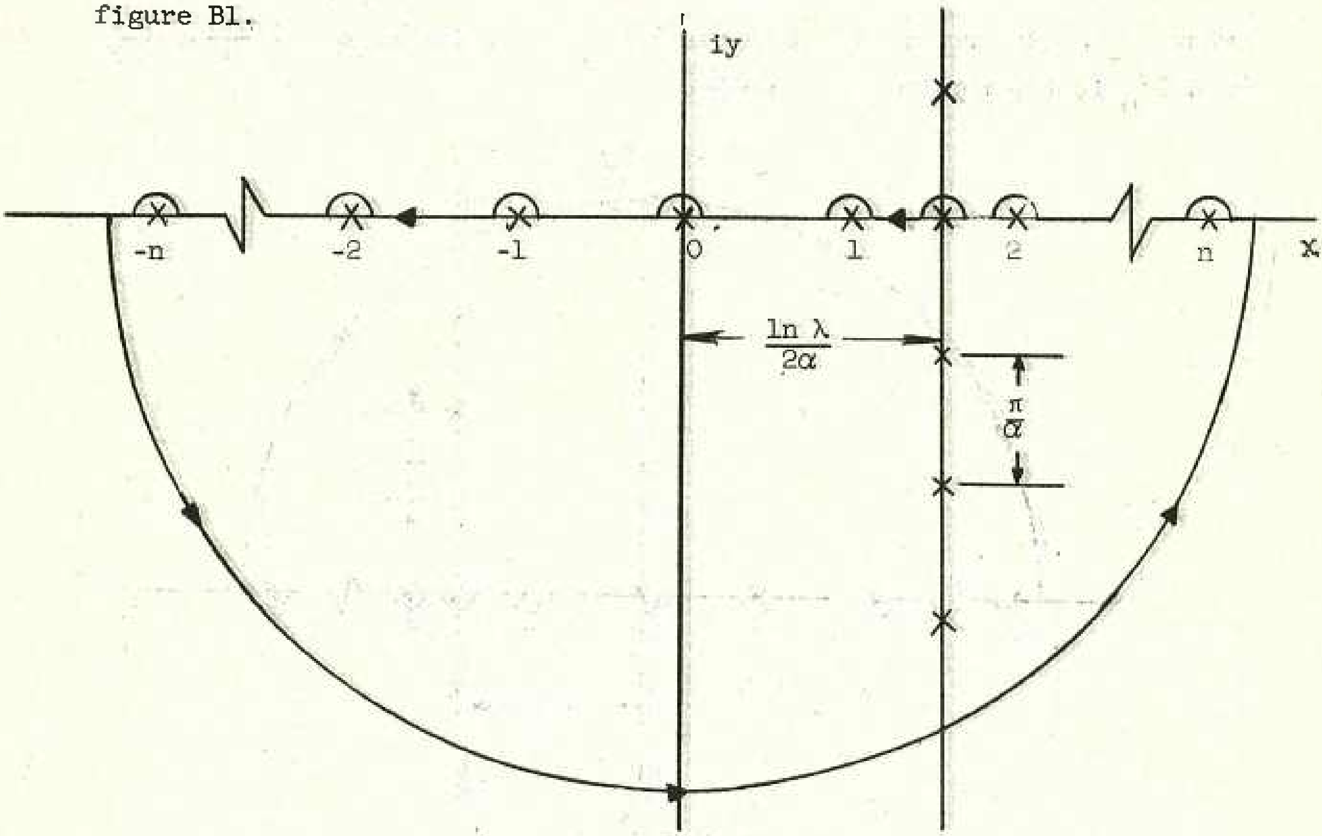

Figure B1

This contour also encircles the poles of the function in the complex plane, i.e., $z = \frac{n\pi}{\alpha} i + \frac{\ln\lambda}{2\alpha}$ with $n < 0$. Letting

$$\rho = \sum_{-\infty}^{\infty} \frac{e^{-\alpha n}}{1-\lambda e^{-2\alpha n}} \text{ then } \int_{\infty}^{-\infty} \frac{e^{-\alpha x}}{1-\lambda e^{-2\alpha x}} \frac{dx}{e^{2\pi i x}-1} + \frac{1}{2}\rho = \rho + \frac{1}{2} a_o + 2\pi i R_-$$

where R_- is the sum of the residues of the function at the poles

$z = \frac{n\pi}{\alpha} i + \frac{\ln\lambda}{2\alpha}$, $n < 0$, and a_o is the residue at $z = \frac{\ln\lambda}{2\alpha}$. Now if we multiply

$\frac{e^{-\alpha z}}{1-\lambda e^{-2\alpha z}}$ by $\frac{1}{1-e^{-2\pi i z}}$ and integrate around an appropriate contour we have

$$\sum_{-\infty}^{\infty} \frac{e^{-\alpha n}}{1-\lambda e^{-2\alpha n}} = \int_{c-} \frac{e^{-\alpha z}}{1-\lambda e^{-2\alpha z}} \frac{dz}{1-e^{-2\pi i z}} .$$

If the contour is chosen as in figure B2, we have

$$\int_{-\infty}^{\infty} \frac{e^{-\alpha x}}{1-\lambda e^{-2\alpha x}} \frac{dx}{1-e^{-2\pi i x}} + \frac{1}{2}\rho = \rho + \frac{1}{2} a'_o + 2\pi i R_+$$

where R_+ is the sum of the residues of the function at $z = -\frac{n\pi i}{\alpha} + \frac{\ln\lambda}{2\alpha}$ and a'_o is the residue at $z = \frac{\ln\lambda}{2\alpha}$.

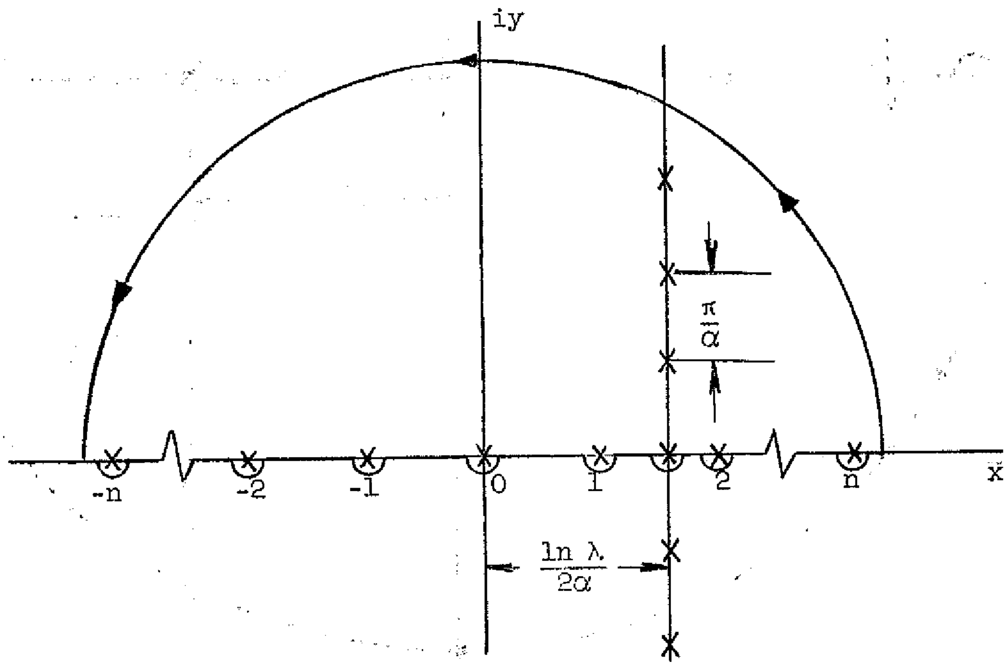

Figure B2

Now if we add the results of these two contours, we have

$$\int_{-\infty}^{\infty} \frac{e^{-\alpha x}}{1-\lambda e^{-2\alpha x}}\,dx \left[\frac{1}{1-e^{-2\pi i x}} - \frac{1}{e^{2\pi i x}-1}\right] + \rho + \frac{1}{2}(a_o + a'_o)$$

$$= 2\rho + 2\pi i\,[R_+ + R_-]$$

or

$$\int_{-\infty}^{\infty} \frac{e^{-\alpha x}}{1-\lambda e^{-2\alpha x}} = \rho + 2\pi i\,[R_+ + R_-] - \frac{1}{2}[a_o + a'_o].$$

Since the integral on the left is zero, we have

$$\rho = \frac{1}{2}[a_o + a'_o] - 2\pi i\,[R_- + R_+]$$

or

$$\sum_{-\infty}^{\infty} \frac{x^n}{1-\lambda x^{2n}} = -\frac{\pi}{2\alpha\sqrt{\lambda}}\,\frac{\sin\theta}{1-\cos\theta} - \frac{\pi}{\alpha\sqrt{\lambda}}\sum_{1}^{\infty}(-1)^n \frac{\sin\theta}{\cosh\frac{2n\pi^2}{\alpha} - \cos\theta}$$

where $\theta = \frac{\pi}{\alpha}\ln\lambda$ and $\lambda > 0$.

The similar series when $\lambda < 0$ can be converted by the same technique to give

$$\sum_{-\infty}^{\infty} \frac{x^n}{1+\lambda x^{2n}} = \frac{\pi}{2\alpha\sqrt{\lambda}} + \frac{\pi}{\alpha\sqrt{\lambda}}\sum_{1}^{\infty} \frac{e^{-(2n+1)\pi^2/\alpha} - \cos\theta}{\cos\theta - \cosh\frac{\pi^2}{\alpha}(2n+1)}$$

To invert the series for equation 7 a different technique is necessary from that used in the preceding section. We consider $\sum_{1}^{\infty} \frac{x^n}{1-x^{2n}}$. This function has a singularity like $\frac{\ln(1-x)}{2\ln x}$ near $x = 1$. We then consider the function

$$S = \sum_{1}^{\infty} \frac{x^n}{1-x^{2n}} - \frac{\ln(1-x)}{\ln x} = \sum_{1}^{\infty} \left(\frac{x^n}{1-x^{2n}} + \frac{x^n}{2n \ln x} \right)$$

$$= \sum_{1}^{\infty} \left(\frac{e^{-\alpha n}}{1-e^{-2\alpha n}} - \frac{e^{-\alpha n}}{2n\alpha} \right)$$

As before we multiply by $\frac{1}{e^{2\pi i z}-1}$ and integrate around a contour inclosing the real axis as shown in figure B3.

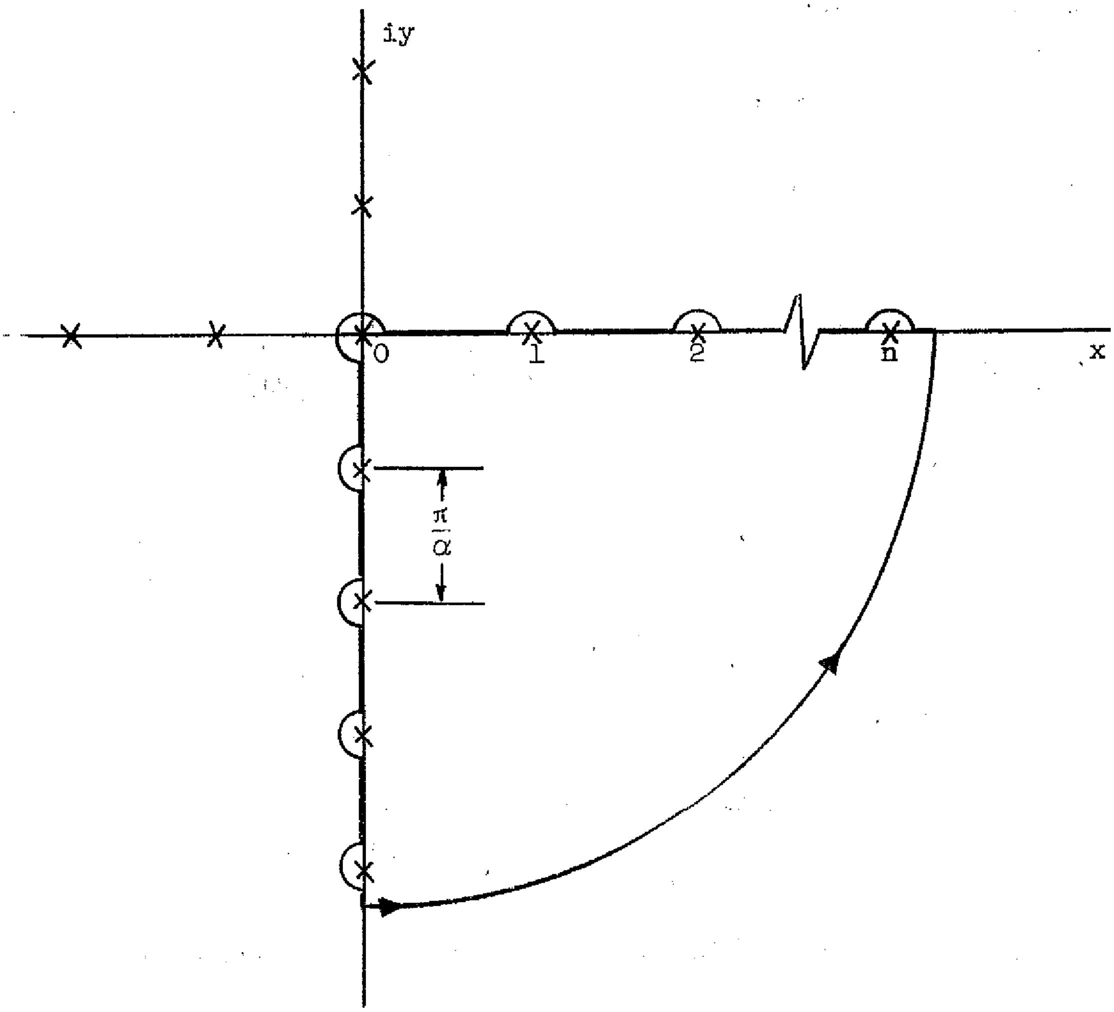

Figure B3

The poles along the imaginary axis are at: $z = -\frac{i\pi n}{\alpha}$.

Then

$$\frac{1}{2} S = \int_{\infty}^{0} \left(\frac{e^{-\alpha x}}{1-e^{-2\alpha x}} - \frac{e^{-\alpha x}}{2\alpha x} \right) \frac{dx}{e^{2\pi i x}-i} + \int_{0}^{-\infty} \left(\frac{e^{-\alpha i y}}{1-e^{-2\alpha i y}} - \frac{e^{-\alpha i y}}{2\alpha i y} \right) \frac{idy}{e^{-2\pi y}-1}$$

$$- \frac{\pi i}{2} R_o - \pi i \sum_{1}^{\infty} R_n$$

where R_n is the residue of $\left[\frac{e^{-\alpha z}}{1-e^{-2\alpha z}} - \frac{e^{-\alpha z}}{2\alpha z} \right] \frac{1}{e^{2\pi i z}-1}$ at $z = -i \frac{n\pi}{\alpha}$.

Now we multiply by $\frac{1}{1-e^{-2\pi i z}}$ and integrate around the contour shown in Figure B4.

Then

$$\frac{1}{2} S = \int_{0}^{\infty} \left(\frac{e^{-\alpha x}}{1-e^{-2\alpha x}} - \frac{e^{-\alpha x}}{2\alpha x} \right) \frac{dx}{1-e^{-2\pi i x}} + \int_{\infty}^{0} \left[\frac{e^{-i\alpha y}}{1-e^{-2i\alpha y}} - \frac{e^{-i\alpha y}}{2i\alpha y} \right] \frac{idy}{1-e^{2\pi y}}$$

$$- \frac{\pi i}{2} \overline{R}_o - \pi i \sum_{1}^{\infty} \overline{R}_n$$

where $\overline{R}_n$ is the residue of $\left[\frac{e^{-\alpha z}}{1-e^{-2\alpha z}} - \frac{e^{-\alpha z}}{2\alpha z} \right] \frac{1}{1-e^{-2\pi i z}}$ at $z = i \frac{n\pi}{\alpha}$.

Adding the two results give

$$S = \int_{0}^{\infty} \frac{e^{-\alpha x}}{1-e^{-2\alpha x}} dx - \int_{0}^{\infty} \frac{\csc \alpha y}{1-e^{2\pi y}} dy + \int_{0}^{\infty} \frac{\cos \alpha y}{\alpha y} \frac{dy}{1-e^{2\pi y}} - \frac{\pi i}{2} (R_o + \overline{R}_o)$$

$$- \int_{0}^{\infty} \frac{e^{-\alpha x}}{2\alpha x} dx - \pi i \sum_{1}^{\infty} (R_n + \overline{R}_n)$$

Now $R_o + \overline{R}_o = \frac{1}{2\pi i}$ and $R_n + \overline{R}_n = 0$.

$$S = \int_{0}^{\infty} \frac{e^{-\alpha x}}{1-e^{-2\alpha x}} dx + \int_{0}^{\infty} \left[\frac{\cos \alpha y}{\alpha y} - \frac{1}{\sin \alpha y} \right] \frac{dy}{1-e^{2\pi y}} - \int_{0}^{\infty} \frac{e^{-\alpha x}}{2\alpha x} dx - \frac{1}{4}$$

or

$$S = \int_{0}^{\infty} \left(\frac{e^{-\alpha x}}{1-e^{-2\alpha x}} - \frac{e^{-\alpha x}}{2\alpha x} \right) dx - \frac{1}{4} + \int_{0}^{\infty} \left(\frac{\cos \alpha y}{\alpha y} - \frac{1}{\sin \alpha y} \right) \frac{dy}{1-e^{2\pi y}}$$

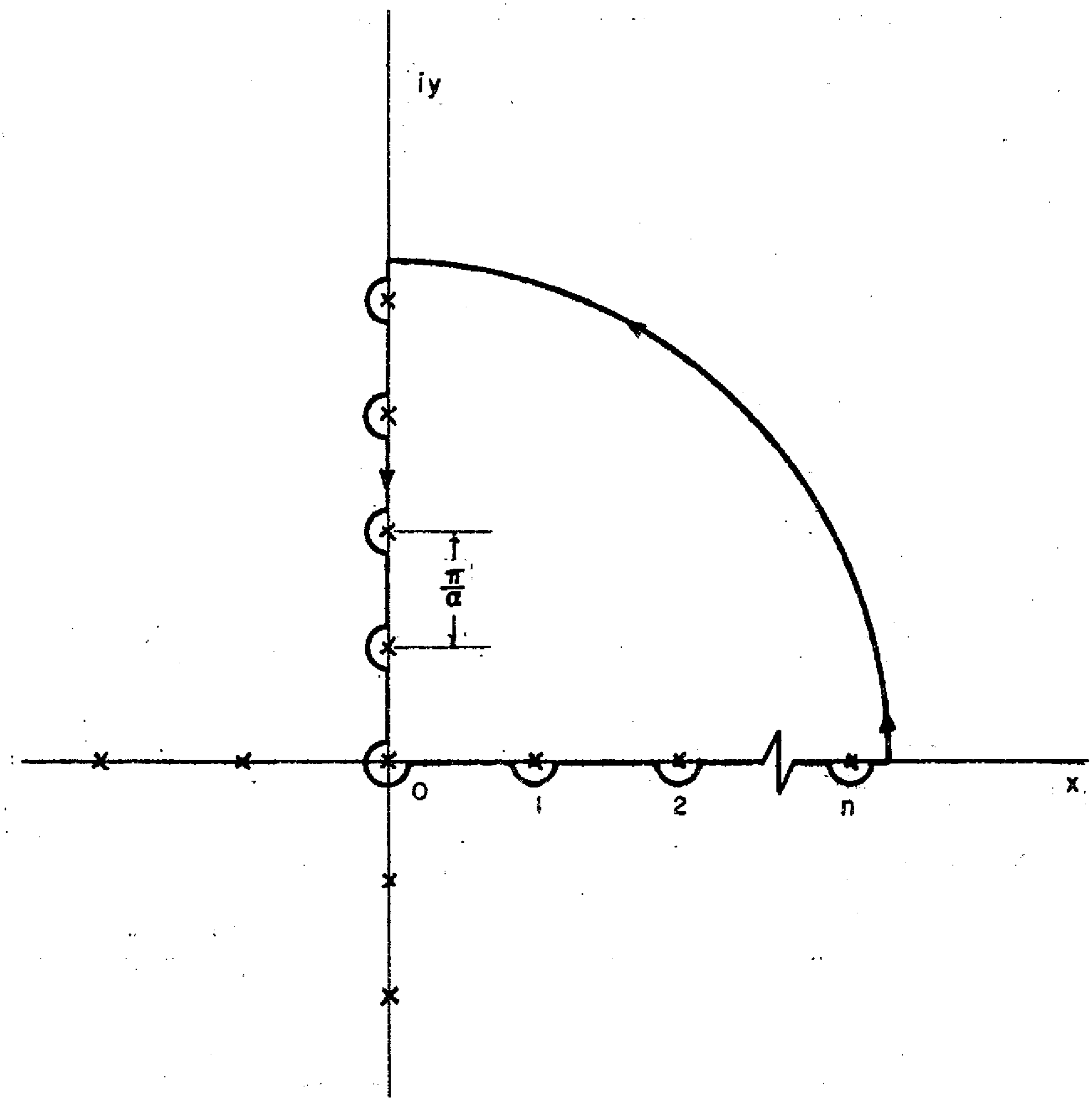

Figure B4

The first integral can be integrated to give

$$\int_0^\infty \left(\frac{e^{-\alpha x}}{1-e^{-2\alpha x}} - \frac{e^{-\alpha x}}{2\alpha x} \right) dx = \frac{1}{2\alpha} [\ln 2 + \gamma]$$

The second integral can be broken up into two parts as

$$\int_0^\infty \left[\left(\frac{1}{\sin \alpha y} - \frac{1}{\alpha y} \right) + \left(\frac{1}{\alpha y} - \frac{\cos \alpha y}{\alpha y} \right) \right] \frac{dy}{e^{2\pi y}-1}$$

The last part can be integrated to give

$$\frac{1}{2\alpha} \left[\ln \left(\frac{\sinh \frac{\alpha}{2}}{\alpha} \right) + \ln 2 \right]$$

The term left can be integrated in series to give

$$\int_0^\infty \left(\frac{1}{\sin \alpha y} - \frac{1}{\alpha y} \right) \frac{dy}{e^{2\pi y}-1} = \sum_1^\infty \frac{(2^{2n-1}-1)}{2(2n)!} (B_{2n-1})^2 \quad \alpha^{2n-1}$$

Then

$$S = \frac{1}{2\alpha} \left\{ \gamma + 2 \ln 2 + \ln \left(\frac{\sinh \frac{\alpha}{2}}{\alpha} \right) \right\} - \frac{1}{4}$$
$$+ \sum_1^\infty \frac{(2^{2n-1}-1)}{2(2n)!} (B_{2n-1})^2 \quad \alpha^{2n-1}$$

where γ = 0.5772157 ...... (Eulers const.)

and B_{2n-1} are Bernoulli numbers.*

Finally

$$\sum_1^\infty \frac{x^n}{1-x^{2n}} = \frac{1}{2\alpha} \left\{ \gamma + \ln 2 - \ln \alpha \right\} + \sum_1^\infty \frac{(2^{2n-1}-1)}{2(2n)!} (B_{2n-1})^2 \quad \alpha^{2n-1}$$

The series involving the Bernoulli numbers is an asymptotic series and can only be used when α is very small, i.e., x near 1. (This is the same result obtained by Schlömilch (ref 3) by a different technique.)

* B. O. Pierce, "A Short Table of Integrals," Ginn and Company, 1929, 3rd Ed., p. 90.

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